We start from Newtonian mechanics, namely the kinetic energy theorem:

$$dE = \vec{F} \cdot d\vec{r} \tag{1}$$

In formula (1) we apply the momentum theorem:

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{2}$$

thus getting the relation:

$$dE = \frac{d\vec{p}}{dt} \cdot d\vec{r} \tag{3}$$

$$dE\,dt - d\vec{p} \cdot d\vec{r} = 0 \tag{4}$$

$$\left(\frac{dE}{c}\vec{e} + d\vec{p}\right) \cdot (c\,dt\,\vec{e} + d\vec{r}) = 0 \tag{5}$$

$$\frac{E}{c}\,\vec{e} + \vec{p} \overset{\text{notation}}{=} \vec{P} \tag{6}$$

$$\boxed{E^2 - p^2c^2 = m_0^2c^4} \tag{7}$$

$$\cosh^2\alpha - \sinh^2\alpha = 1 \tag{8}$$

$$\underbrace{\frac{E^2}{m_0^2c^4}}_{=\cosh^2\alpha} - \underbrace{\frac{p^2}{m_0^2c^2}}_{=\sinh^2\alpha} = 1 \tag{9}$$

$$\begin{cases} E = m_0c^2\cosh\alpha & (10) \\ p = m_0c\sinh\alpha & (11) \end{cases}$$

Case $v = c\tanh\beta < c$ **subluminal velocity**

$$\left(m_0c^2\sinh\alpha\,d\alpha\,\vec{e} + m_0c^2\cosh\alpha\,d\alpha\,\vec{i}\right) \cdot \left(c\tau\cosh\beta\,\vec{e} + c\tau\sinh\beta\,\vec{i}\right) = 0 \tag{12}$$

$$\left(\tanh\alpha\,\vec{e} + \vec{i}\right) \cdot \left(\vec{e} + \tanh\beta\,\vec{i}\right) = 0 \tag{13}$$

$$\vec{e} \cdot \vec{i} = 0; \quad \vec{e}^{\,2} = -1; \quad \vec{i}^{\,2} = 1 \tag{14}$$

$$-\tanh\beta + \tanh\alpha = 0 \tag{15}$$

$$\boxed{\beta = \alpha} \tag{16}$$

$$\begin{cases} \cosh\alpha = \dfrac{1}{\sqrt{1 - \tanh^2\alpha}} = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} & (17) \\ \sinh\alpha = \dfrac{\tanh\alpha}{\sqrt{1 - \tanh^2\alpha}} = \dfrac{\dfrac{v}{c}}{\sqrt{1 - \dfrac{v^2}{c^2}}} & (18) \end{cases}$$

$$\mathbf{E} = \mathbf{m_0c^2}\cosh\alpha = \frac{\mathbf{m_0c^2}}{\sqrt{\mathbf{1} - \dfrac{\mathbf{v^2}}{\mathbf{c^2}}}} \tag{19}$$

$$\mathbf{p} = \mathbf{m_0c}\sinh\alpha = \frac{\mathbf{m_0v}}{\sqrt{\mathbf{1} - \dfrac{\mathbf{v^2}}{\mathbf{c^2}}}} \tag{20}$$

Case $v = c\coth\beta > c$ **superluminal velocity**

$$\begin{cases} E = cP\sinh\alpha & (21) \\ p = P\cosh\alpha & (22) \end{cases}$$

$$\left(P\cosh\alpha\,d\alpha\,\vec{e} + P\sinh\alpha\,d\alpha\,\vec{i}\right) \cdot \left(l\sin\beta\,\vec{e} + l\cosh\beta\,\vec{i}\right) = 0 \tag{23}$$

$$\left(\coth\alpha\,\vec{e} + \vec{i}\right) \cdot \left(\vec{e} + \coth\beta\,\vec{i}\right) = 0 \tag{24}$$

$$\vec{e} \cdot \vec{i} = 0; \quad \vec{e}^{\,2} = -1; \quad \vec{i}^{\,2} = 1 \tag{25}$$

$$-\coth\alpha + \coth\beta = 0 \tag{26}$$

$$\boxed{\beta = \alpha} \tag{27}$$

$$\begin{cases} \cosh\alpha = \dfrac{\coth\alpha}{\sqrt{\coth^2\alpha - 1}} = \dfrac{\dfrac{v}{c}}{\sqrt{\dfrac{v^2}{c^2} - 1}} & (28) \\ \sinh\alpha = \dfrac{1}{\sqrt{\coth^2\alpha - 1}} = \dfrac{1}{\sqrt{\dfrac{v^2}{c^2} - 1}} & (29) \end{cases}$$

$$\mathbf{E} = \mathbf{Pc}\sinh\alpha = \frac{\mathbf{Pc}}{\sqrt{\dfrac{\mathbf{v^2}}{\mathbf{c^2}} - \mathbf{1}}} \tag{30}$$

$$\mathbf{p} = \mathbf{P}\cosh\alpha = \frac{\mathbf{P}\dfrac{\mathbf{v}}{\mathbf{c}}}{\sqrt{\dfrac{\mathbf{v^2}}{\mathbf{c^2}} - \mathbf{1}}} \tag{31}$$