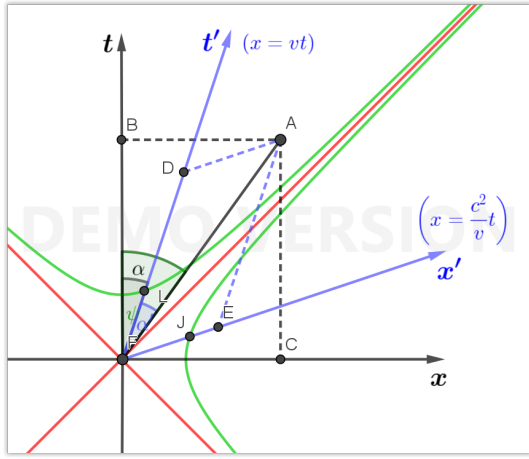




$$c = 1; \quad v < 1$$

↓

$$\begin{cases} t^2 - x^2 = t'^2 - x'^2 = \tau^2 \\ v = \tanh \psi; \quad u = \frac{x}{t} = \tanh \alpha; \\ u' = \frac{x'}{t'} = \tanh \alpha' \\ \alpha' = \alpha - \psi \end{cases}$$

$$c = 1; \quad v > 1$$

↓

$$\begin{cases} x^2 - t^2 = x'^2 - t'^2 = l^2 \\ v = \coth \psi; \quad u = \frac{x}{t} = \coth \alpha; \\ u' = \frac{x'}{t'} = \coth \alpha' \\ \alpha' = \alpha - \psi \end{cases}$$

$$\begin{cases} t' = t \cosh \psi - x \sinh \psi \\ x' = -t \sinh \psi + x \cosh \psi \end{cases}$$

$$u = \frac{x}{t}; \quad u' = \frac{x'}{t'} \quad (15)$$

$$u' = \frac{u - v}{1 - uv} \quad (16)$$

$$\begin{aligned} \coth \alpha' &= \frac{\coth \alpha - \tanh \psi}{1 - \coth \alpha \tanh \psi} \\ &= \frac{1 - \coth \psi \coth \alpha}{\coth \alpha - \coth \psi} = \coth(\alpha - \psi) \end{aligned} \quad (17)$$

$$v = \coth \psi \quad (18)$$

$$\begin{cases} \cosh \psi = \frac{\coth \psi}{\sqrt{\coth^2 \psi - 1}} = \frac{v}{\sqrt{v^2 - 1}}; \\ \sinh \psi = \frac{1}{\sqrt{\coth^2 \psi - 1}} = \frac{1}{\sqrt{v^2 - 1}} \end{cases} \quad (19)$$

$$\begin{cases} \sinh \psi = \frac{1}{\sqrt{\coth^2 \psi - 1}} = \frac{1}{\sqrt{v^2 - 1}} \end{cases} \quad (20)$$

The corresponding coordinates transformation is:

$$\begin{cases} t' = \frac{vt - x}{\sqrt{v^2 - 1}} = \frac{\frac{v}{c}t - \frac{x}{c}}{\sqrt{\frac{v^2}{c^2} - 1}} \end{cases} \quad (21)$$

$$\begin{cases} x' = \frac{-ct + \frac{v}{c}x}{\sqrt{\frac{v^2}{c^2} - 1}} \end{cases} \quad (22)$$

$$\begin{cases} t' = \frac{t - \frac{x}{v}}{\sqrt{1 - \frac{c^2}{v^2}}} \\ x' = \frac{x - \frac{c^2}{v}t}{\sqrt{1 - \frac{c^2}{v^2}}} \end{cases} \quad (23)$$

$$\begin{cases} x' = \frac{x - \frac{c^2}{v}t}{\sqrt{1 - \frac{c^2}{v^2}}} \end{cases} \quad (24)$$

Please take a look at the following animation:

(13) <https://youtu.be/vSW1EoS9Hzw>

(14) <https://youtu.be/DDwfLImsS0o>