Let be a relativistic particle with a universe line in the space-time diagram. The condition that its 3-dimensional velocity is less than the speed of light c is very restrictive. This condition is sufficient to respect causality, i.e. that the effect takes place temporally after the cause in all reference systems, it is a mathematically sufficient condition but it is not necessary. The universe line of the particle has an equation

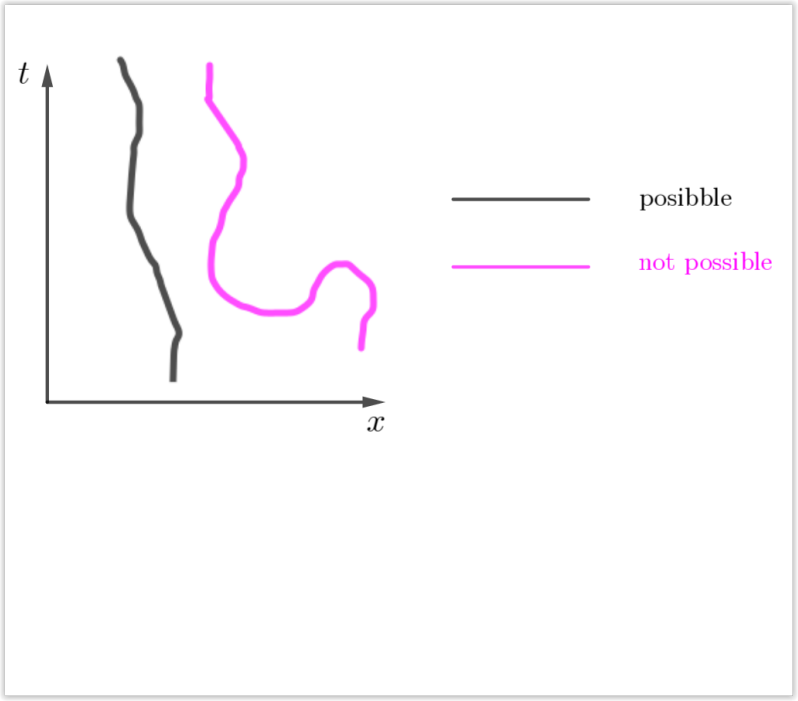
$$x = f(t) \tag{1}$$

The elementary condition for f to be a function is the following:

$$t_1 = t_2 \rightarrow f(t_1) = f(t_2) \forall t_1,\ t_2 \tag{2}$$

$$x_2 - x_1 = \Delta x = f(t_2) - f(t_1) = f(t_1 + \Delta t) - f(t_1) \tag{3}$$

$$\Delta t = 0 \Rightarrow \Delta x = 0 \tag{4}$$



(see figure)
If this condition is satisfied in one reference system, then it is satisfied in all reference systems. Here is the proof for inertial systems:

$$\begin{cases} \Delta t' = \Delta t \cosh\psi - \Delta x \sinh\psi & (5) \\ \Delta x' = -\Delta t \sinh\psi + \Delta x \cosh\psi & (6) \end{cases}$$

All coordinate variations cancel simultaneously.