

Let's apply these to spacetime endowed with the Minkowski metric, as in the figure. We consider all physical quantities divided by their unit of measure. For example for SI:

$$v \leftarrow \frac{v}{[m/s]}, \ t \leftarrow \frac{t}{[s]}, \ x \leftarrow \frac{x}{[m]}$$
 (1)

$$\int t^2 - x^2 = 1 (2)$$

$$\begin{cases} t^{2} - x^{2} = 1 & (2) \\ x = \frac{v}{c}t & (3) \\ x = x_{D} & (4) \\ t = t_{D} & (5) \end{cases}$$

$$x = x_D \tag{4}$$

$$t = t_D \tag{5}$$

$$\begin{cases} t_D^2 - \frac{v^2}{c^2} t_D^2 = 1 \\ OE = \frac{t_E}{t_D} \end{cases}$$
 (6)

$$OE = \frac{t_E}{t_D} \tag{7}$$

$$\begin{cases} t_D = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ OE^2 = \left(1 - \frac{v^2}{c^2}\right) t_E^2 \end{cases}$$
 (8)

$$OE^2 = \left(1 - \frac{v^2}{c^2}\right)t_E^2\tag{9}$$

$$OE^2 = t_E^2 - \frac{v^2}{c^2} t_E^2 = t_E^2 - x_E^2$$
 (10)

Q.E.D.