Clocks synchronization in spacetime

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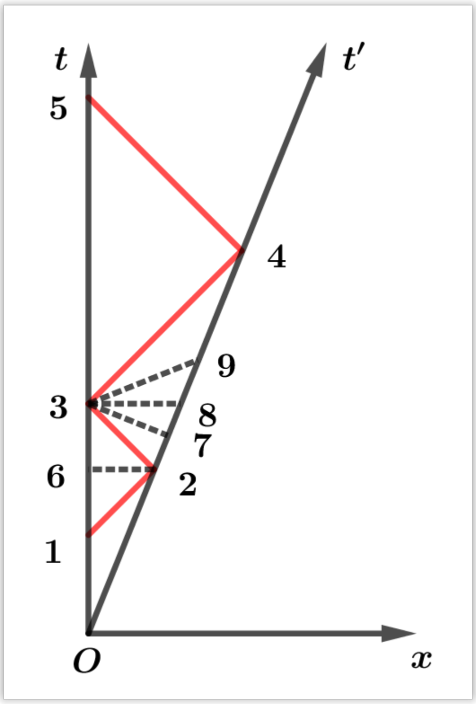

The physical situation consists of two observers S and S' which are in rectilinear and uniform translational motion relative to each other. The x and x' axes coincide and the motion is along them in 3-D. Each observer has a clock located at the origin of the 3-D axes. The observers emit electromagnetic TV signals containing the image of their own clock and re-emit the received image of the other. Thus, observer S' has in event 4 both its own time at event 2 and S's time at event 3. The question arises as to what condition the three time coordinates must satisfy in order for the induced structure in spacetime to be Euclidean. For this we write the equations of some lines, and the intersection of two lines is an event.

$$\begin{cases} x = -c(t - t_3) & (1) \\ x = vt & (2) \\ t = t_2 & (3) \\ x = x_2 & (4) \end{cases}$$

$$\begin{cases} vt_2 = -ct_2 + ct_3 & (5) \\ x_2 = vt_2 & (6) \end{cases}$$

$$\begin{cases} ct_2 = \dfrac{ct_3}{1 + \dfrac{v}{c}} & (7) \\ x_2 = vt_2 & (8) \end{cases}$$

$$c'^2 t_2'^2 = c^2 t_2^2 + x_2^2 = (c^2 + v^2) c^2 t_2^2 \tag{9}$$

$$\begin{cases} ct_2' = \dfrac{t_3\sqrt{c^2 + v^2}}{1 + \dfrac{v}{c}} & (10) \\ x_2' = 0 & (11) \end{cases}$$

$$\begin{cases} x - x_3 = c(t - t_3) & (12) \\ x = vt & (13) \\ t = t_4 & (14) \\ x = x_4 & (15) \end{cases}$$

$$\begin{cases} ct_4 = \dfrac{ct_3}{1 - \dfrac{v}{c}} & (16) \\ x_4 = vt_4 & (17) \end{cases}$$

$$\begin{cases} ct_4' = \dfrac{t_3\sqrt{c^2 + v^2}}{1 - \dfrac{v}{c}} & (18) \\ x_4' = 0 & (19) \end{cases}$$

$$\begin{cases} x - x_3 = -\dfrac{c^2}{v}(t - t_3) & (20) \\ x = vt & (21) \\ t = t_7 & (22) \\ x = x_7 & (23) \end{cases}$$

$$\begin{cases} (c^2 + v^2)t = c^2 t_3 & (24) \\ x = vt & (25) \\ t = t_7 & (26) \\ x = x_7 & (27) \end{cases}$$

$$\begin{cases} ct_4' = \dfrac{t_3\sqrt{c^2 + v^2}}{1 - \dfrac{v}{c}} t_7 = \dfrac{c^2 t_3}{c^2 + v^2} & (28) \\ x_7 = vt_7 & (29) \end{cases}$$

$$ct_7' = \frac{ct_3}{\sqrt{1 + \dfrac{v^2}{c^2}}} \tag{30}$$

$$ct'_2 = \frac{t_3\sqrt{c^2+v^2}}{1+\frac{v}{c}} \tag{31}$$

$$ct'_4 = \frac{t_3\sqrt{c^2+v^2}}{1-\frac{v}{c}} \tag{32}$$

$$ct'_7 = \frac{ct_3}{\sqrt{1+\frac{v^2}{c^2}}} \tag{33}$$

$$37^2 = c^2t_3^2 - c'^2t_7'^2 = c^2t_3^2 - \frac{c^2t_3^2}{1+\frac{v^2}{c^2}} = \frac{v^2t_3^2}{1+\frac{v^2}{c^2}} \tag{34}$$

$$37 = \frac{vt_3}{\sqrt{1+\frac{v^2}{c^2}}} \tag{35}$$

$$27 = c(t'_7 - t'_2) = \frac{ct_3}{\sqrt{1+\frac{v^2}{c^2}}} - \frac{t_3\sqrt{c^2+v^2}}{1+\frac{v}{c}} \tag{36}$$

$$= \frac{1+\frac{v}{c}-1-\frac{v^2}{c^2}}{\left(1+\frac{v}{c}\right)\sqrt{1+\frac{v^2}{c^2}}}ct_3 = \frac{vt_3\left(1-\frac{v}{c}\right)}{\left(1+\frac{v}{c}\right)\sqrt{1+\frac{v^2}{c^2}}} \tag{37}$$

$$\frac{c_-}{c} = \frac{\frac{vt_3}{\sqrt{1+\frac{v^2}{c^2}}}}{\frac{vt_3\left(1-\frac{v}{c}\right)}{\left(1+\frac{v}{c}\right)\sqrt{1+\frac{v^2}{c^2}}}} = \frac{1+\frac{v}{c}}{1-\frac{v}{c}} \tag{38}$$

$$c(t'_4 - t'_7) = \frac{t_3\sqrt{c^2+v^2}}{1-\frac{v}{c}} - \frac{ct_3}{\sqrt{1+\frac{v^2}{c^2}}} = \frac{vt_3\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)\sqrt{1+\frac{v^2}{c^2}}} \tag{39}$$

$$\frac{c_+}{c} = \frac{1-\frac{v}{c}}{1+\frac{v}{c}} \tag{40}$$

$$c_+c_- = c^2 \tag{41}$$

$$\frac{t'_4}{t'_2} = \frac{c+v}{c-v} \tag{42}$$

$$\frac{t'_4 - t'_2}{t'_4 + t'_2} = \frac{(c+v)-(c-v)}{(c+v)+(c-v)} = \frac{v}{c} \tag{43}$$

$$1+\frac{v^2}{c^2}=1+\left(\frac{t'_4-t'_2}{t'_4+t'_2}\right)^2=\frac{t'^2_4+2t'_2t'_4+t'^2_2+t'^2_4-2t'_2t'_4+t'^2_2}{(t'_4+t'_2)^2}=\frac{2(t'^2_4+t'^2_2)}{(t'_4+t'_2)^2} \tag{44}$$

$$1-\frac{v^2}{c^2}=1-\left(\frac{t'_4-t'_2}{t'_4+t'_2}\right)^2=\frac{t'^2_4+2t'_2t'_4+t'^2_2-t'^2_4+2t'_2t'_4-t'^2_2}{(t'_4+t'_2)^2}=\frac{4t'_4t'_2}{(t'_4+t'_2)^2} \tag{45}$$

$$\frac{t'_2t'_4}{t_3^2}=\frac{1+\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}=\frac{t'^2_4+t'^2_2}{2t'_2t'_4} \tag{46}$$

$$\frac{t'_2t'_4}{t_3}\sqrt{2}=\sqrt{t'^2_4+t'^2_2} \tag{47}$$

$$\boxed{\frac{2}{t_3^2}=\frac{1}{t'^2_2}+\frac{1}{t'^2_4}} \tag{48}$$

Ecuation (48) must be satisfied in order of coordinates to be euclidian. We note that the spacetime is the same. In the case of Minkowskian spacetime the ecuation is $t_3^2 = t'_2t'_4$. In Minkowskian space the upper limit of velocities should be interpreted as the limit at which the hyperbolic structure of the geometry can be used.
Clock synchronization leads to the parameterization of the t' axis, and is done by manipulating the physical clock of observer S' initially in event 4.