

Unfortunately, the length contraction of the theory of special relativity is still not formulated precisely enough. The observer on the train and the one in the tunnel are not the only actors involved. It is only this fatal inaccuracy that causes the train-in-tunnel paradox.

Another observer is involved, namely the one to whom the doors at the tunnel's ends are simultaneously closed. This third observer may be at rest either in relation to the train or to the tunnel, or may have some intermediate speed. So this third observer defines the simultaneity at the tunnel ends.

If the third observer is at rest with respect to the tunnel, then the train will be shorter than the tunnel and as such the tunnel is longer than the train.

If the third observer is at rest relative to the train, then the train is longer than the tunnel, and as such the tunnel is shorter than the train.

There is an intermediate speed for the third observer, for which the train and tunnel lengths are equal. Let $v = c \tanh \alpha$ be the speed of the train relative to the tunnel, and $v_{1/2}$ the speed of the third observer relative to the tunnel. It can easily be shown that:

$$v_{1/2} = c \tanh \frac{\alpha}{2} \tag{1}$$

Please watch the following animations.

<https://youtu.be/EIIcI2wz014>

https://youtu.be/G_MLBVmJToM

<https://youtu.be/sZcIMZG1M1o>